Math 30-1 Unit 3 Exponents and Logarithms Review Name _______________

1. Determine the following characteristics of the graph of $f(x)=4(10^x)+3$

   Domain $\{x \mid x \in R\}$
   Range $\{y \mid y \geq 3, y \in R\}$
   Equation of the asymptote $y = 3$
   Coordinates of the y-intercept $x=0, y=3$

2. Given $y=4^x$ and its inverse $x=4^y \Leftrightarrow y=\log_4 x$.
   a) Complete the table of values for each

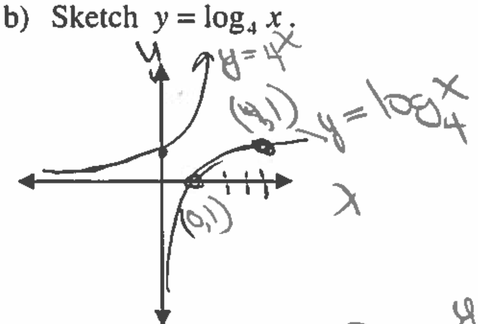
$y=4^x$

x	y
-2	$\frac{1}{16}$
-1	$\frac{1}{4}$
0	1
1	4
2	16

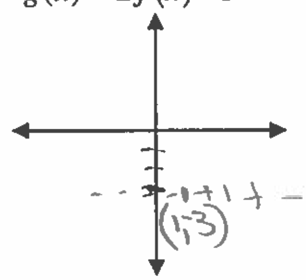
$y=\log_4 x$

x	y
$\frac{1}{16}$	-2
$\frac{1}{4}$	-1
1	0
4	1
16	2

b) Sketch $y=\log_4 x$.



c) If $f(x)=\log_4 x$, sketch $g(x)=2f(x)-3$



3. Given $y=\log_2(x-4)$ ∴ $2^y = x-4$
   $2^y+4 = x$ → Inverse $2^x+4 = y$
   a) Determine the following characteristics
   Domain $\{x \mid x>4, x \in R\}$
   Range $y \in R$
   Equation of the asymptote $x=4$
   Coordinates of the x-intercept $x=5$
   $y=0$
   $2^0 = x-4$ ∴ $x=5$

   b) If $f(x)=\log_2 x$, what is the x-intercept of $h(x)=f\left(\frac{1}{3}x\right)$?
   $(3,0)$

4. The point (1, 0) is on the graph of $\boxed{y=\log_3 x}$. Determine the coordinates of the image point after the graph has been translated according to the equation $y=\log_3(2x-4)$.

   H.S. by $\frac{1}{2}$ Right 2 $y=\log_3(2[x-2])$
   $(1,0) \longrightarrow \left(\frac{1}{2},0\right) \rightarrow \left(\frac{5}{2},0\right)$

5. Describe in words the transformations required to transform the graph of $y=\log_5 x$ to the graph of $y=-\log_5\left(\frac{1}{2}x+6\right)-3$. $= -\log_5\left[\frac{1}{2}[x+12]\right]-3$

   S: H.S. by 2
   R: Reflect in x axis
   T: 12 left, 3 Down

10. Express $\log_2 200$ in terms of x given $x = \log_2 5$

$2^x = 5$ $\quad$ $\log_2 200 = \log_2(25 \cdot 8) = \log_2 25 + \log_2 8$

$= 2\log_2 5 + 3$ $\quad$ $2x+3$

11. Given $3 = \log_x 8$, evaluate $\log_x 32$.

$x^3 = 8$ $\quad$ $x = 2$ $\quad$ $\log_2 32 = 5$

12. Using the equation $\log_{27} x = y$, what would the expression $\log_9 x$ equal?

Change Base

$\log_{27} x = \dfrac{\frac{\log x}{\log 9}}{\frac{\log 27}{\log 9}} = \dfrac{\log 9}{\log 27} \cdot \dfrac{\log x}{\log 9}$

$\frac{2}{3} \log_9 x$

13. If $\log_3 5 = x$, express $2\log_3 45 - \frac{1}{2}\log_3 225$ in terms of x.

14. For each of the following, solve for x. Remember to list the domain.

a) $\log_2 x - \log_2(x+2) = 3$ $\qquad$ b) $\log x + \log 6 = \log\frac{1}{2}$

c) $2(\log_9 x)^2 - \log_9 x^7 - 4 = 0$ $\qquad$ d) $5^{2x-1} = 3^{x+2}$

15. Solve for the value of the variable. $3^{(2x+1)} = \left(\frac{1}{5}\right)^{(x-3)}$

16. A bacteria culture starts with 5000 bacteria. After 6 hours, the estimated count is 80,000. What is the doubling period for this bacteria culture?

17. The population of a city is increasing at a constant rate of 5% per year. The city's present population is 200 000. Determine the minimum number of years it will take for the population to exceed 500 000.

(12)

$$\log_{27} x = y$$

$$\frac{\frac{\log x}{\log 9}}{\frac{\log 27}{\log 9}} = y$$

$$\frac{\log 9}{\log 27} \log_9 x = y$$

$$\log_{27} 9 \log_9 x = y$$

$$\left(\frac{2}{3}\right) \log_9 x = y$$

$$\log_9 x = \boxed{\frac{3}{2} y}$$

☺

∴ $27^y = x$

$$\log_9 x$$

$$\log_9 \left(27^y\right)$$

$$y \log_9 27$$

$$\boxed{y\left(\frac{3}{2}\right)}$$

(13) $3^x = 5$ $\log_3 5 = x$

$\log_3 (9 \cdot 5)^2 - \log_3 (225)^{\frac{1}{2}}$

$\log_3 81 \cdot 5^2 - \log_3 15$

$\log_3 81 + 2\log_3 5 - [\log_3 5 + \log_3 3]$

$4 + 2x - x - 1$

$x + 3$

14 a) $\log_2 \left[\frac{x}{x+2}\right] = 3$

$\frac{8}{1} = \frac{x}{x+2}$ $x \geq -2$

$8x + 16 = x$

$7x = -16$

$x = \frac{-16}{7}$ ✗

No S.

b) $\log(6x) = \log(\frac{1}{2})$ $x > 0$

$6x = \frac{1}{2}$

$x = 12$

c) $\log_a x = b$

$2b^2 - 7b - 4 = 0$

$(2b + 1)(b - 4)$

$b = -\frac{1}{2}$ $b = 4$

$\log_a x = -\frac{1}{2}$ $\log_a x = 4$

$x > 0$

$x = \frac{1}{3}$ $x = 6561$

(d) $(2x-1)\log 5 = (x+2)\log 3 \quad \{x \in \mathbb{R}\}$

$(\log 5)2x - \log 5 = (\log 3)(x) + \log 3$

$[2\log 5 - \log 3]x = \log 3 + \log 5$

$$x = \frac{\log 3 + \log 5}{2\log 5 - \log 3}$$

(15) $(2x+1)(\log 3) = (x-3)(\log \frac{1}{5})$

$\log 3\, 2x + \log 3 = (\log \frac{1}{5})x - 3(\log \frac{1}{5})$

$[2\log 3 - \log \frac{1}{5}]x = 3\log \frac{1}{5} - \log 3$

$$x = \frac{3\log \frac{1}{5} - \log 3}{2\log 3 - \log \frac{1}{5}}$$

(16) $80000 = 5000\, 2^{\frac{6}{d}}$

$16 = 2^{\frac{6}{d}}$

$4 = \frac{6}{d}$

$d = \frac{6}{4} = 1.5$

17 $500{,}000 = 200{,}000\,(1.05)^x$

$2.5 = (1.05)^x$

$\log 2.5 = x\log(1.05)$

$18.8 = x$

$\therefore$ 19 years

18 $[H^+] = 10^{-2.3}$ lemon 2.3 ☺ $\qquad 10^{-pH} = H^+$

$[H^+] = 10^{-6.3}$ milk

lemon more acidic ☺

19 $pH = -3.49$

20 $10^{4.7} = 50118.7$ times

21 $L_{dB} = 10\log\left[\frac{10^{-5}}{10^{-12}}\right]$

$= 10\log[10^7]$

$= 70$ db ☺

22 $\log 8 = .903$

$\therefore \uparrow$ in decibel $= 9$ db

31 db to the

New effectiveness